Determining the number of factors: the example of the NEO-PI-R

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The number of factors to extract from a data set is a long time problem in psychometrics. Henry Kaiser described it as easy and one that he solved everyday before breakfast. But the problem was to find the right solution (Horn and Engstrom, 1979). Here I examine the number of factors to extract from a well defined data set that supposedly has a "known" answer. Unfortunately, each of four different procedures gives a different answer.

The correlation matrix examined is that of the 30 facets of the NEO-PI-R, taken from the test manual of the NEO. (The matrix was kindly supplied by Suitbert Ertel to the readers of the IDANET newsgroup.) The number of cases is 1,000.

1 Parallel analysis and the Scree Test

One of the most commonly used procedures is simply to plot the eigen values of the first N principal components and look for sharp breaks in the plot. Cattell's scree test is analogically finding where a mountain starts after climbing up the pile of rubble (the scree) that has fallen off (Cattell, 1966).

Parallel Analysis is an alternative to the scree test and compares the eigen values of the observed data to those of a random data set of the same size.

Both of these tests are implemented in the `fa.parallel` function in the *psych* package in R.

The solution shown in Figure 1 suggests that five is the most appropriate number of factors. This is, of course, what one would expect from the definitive measure for the "Five Factor Theory'.

2 Chi Square goodness of fit estimates

Some advocate that factors should be extracted until the correlations in the residual matrix do not differ from what would be expected by chance. Using Maximum Likelihood estimates of the factor structure, we can apply either Bartlett's test or the Lawley and Maxwell test

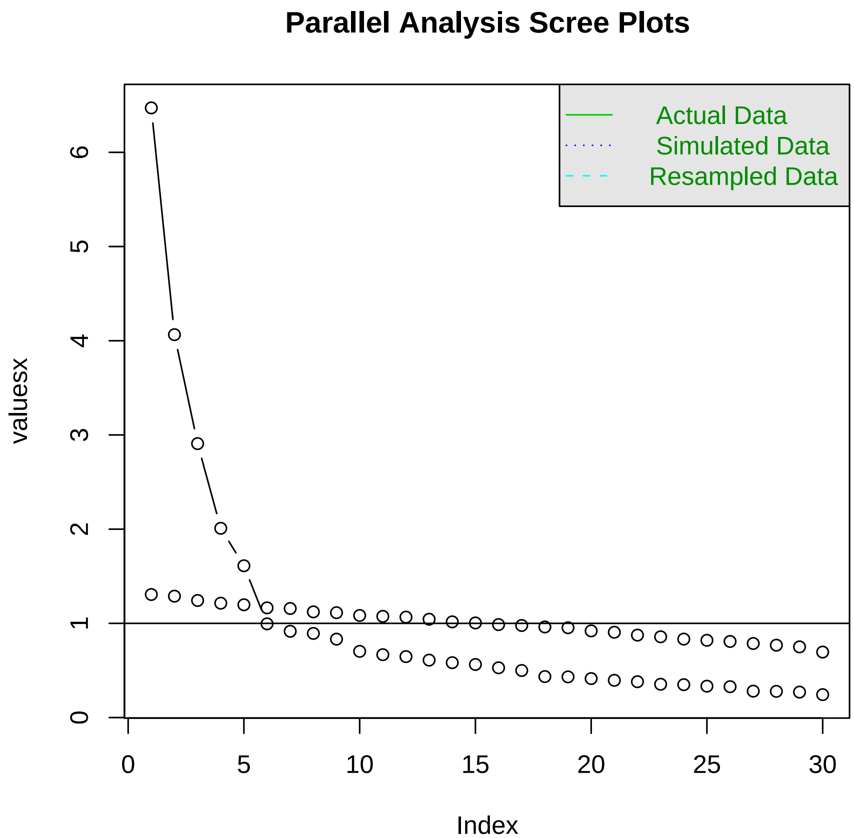

Figure 1: The scree test and parallel analyses of the NEO-PI-R suggest that five factors are most appropriate. This is, of course, the expected solution.

and then show the χ^2 values as a function of the number of factors extracted. Unfortunately, this procedure leads to the interesting phenomena of extracting more factors the larger the sample size. For this data set, with 1,000 subjects, the χ^2 show that the residuals are greater than a 0 matrix for up to 15 factors.

Table 1: The χ^2 test for the number of factors tests the probability that the residual matrix does not differ from a chance matrix. Being very sensitive to sample size, this test will tend to overfactor for large samples.

```
> vss.neo16.mle <- VSS(neo.df, 16, pc = "mle", n.obs = 1000)
> round(vss.neo16.mle[, 1:3], 2)
```

```
   dof   chisq prob
1  405 7996.56 0.00
2  376 5517.28 0.00
3  348 3514.73 0.00
4  321 2113.94 0.00
5  295 1443.45 0.00
6  270 1150.59 0.00
7  246  897.84 0.00
8  223  709.15 0.00
9  201  579.26 0.00
10 180  486.38 0.00
11 160  387.71 0.00
12 141  310.90 0.00
13 123  233.19 0.00
14 106  168.11 0.00
15  90  123.64 0.01
16  75   89.96 0.11
```

3 Very Simple Structure

Yet another way to choose the number of factors is to use the Very Simple Structure (VSS) criterion (Revelle and Rocklin, 1979). VSS recognizes that most users of factor analysis tend to interpret factor output by focusing their attention on the largest loadings for every variable and ignoring the smaller ones. (And in fact score the scales based not upon factor loading estimates but rather just unit weighted composites of these salient variables.) Very Simple Structure operationalizes this tendency by comparing the original correlation matrix to that reproduced by a simplified version (S_{ck}) of the original factor matrix (F). S_{ck} is composed of just the c greatest (in absolute value) loadings for each variable and k is the number of factors. C (or complexity) is a parameter of the model and may vary from 1 to the number of factors. Essentially this is a simple way of doing confirmatory factor analysis.

The basic factor model is

$$R \approx FF' + U^2 \tag{1}$$

where U^2 is a diagonal matrix of uniquenesses.

The Very Simple Structure (VSS) model is

$$R \approx SS' + U^2 \tag{2}$$

with residuals

$$R^* = R - (SS' + U^2) \tag{3}$$

and the VSS criterion is

$$VSS_{ck} = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{*2} / \sum_{j=1}^{n}\sum_{i=1}^{n} r_{ij}^{2} \tag{4}$$

VSS for a given complexity will tend to peak at the optimal (most interpretable) number of factors (Revelle and Rocklin, 1979). Simulations have shown that if the data are simple structured, and each variable is of complexity one (i.e., has only one non-zero loading), then either under or over factoring will lead to a worse solution than extracting the correct number of factors. Many personality scales are somewhat more complex in that some items are of complexity two (e.g., Hofstee et al. (1992)). In this case, the VSS criterion for complexity two will indicate the optimal number of interpretable factors.

4 Hierarchical Cluster Analysis and the Internal Structure of Tests

An alternative to factor analysis for examining the internal structure of tests and to determine the optimal number of scales to derive from a multidimensional battery is hierarchical cluster analysis using the ICLUST algorithm (Revelle, 1979). Scales are combined until β, the worst split half reliability coefficient, fails to increase. Revelle (1979) suggested that β is an estimate of the general factor saturation of a test. Although Zinbarg et al. (2005) showed that McDonald's hierachical ω is a better estimate of the general factor saturation, it is still the case that using the β coefficient as a stopping criterion in hierachical clustering produces very interpretable scales. ICLUST is implemented in the `ICLUST` and `ICLUST.rgraph` functions in the *psych* package in R.

Two of these four clusters are moderately intercorrelated (r = .41) suggesting that a three cluster solution might be better (Table 2, Figure 4.

```
> vss.neo.mle <- VSS(neo.df, pc = "mle", n.obs = 1000)
> VSS.plot(vss.neo.mle)
```

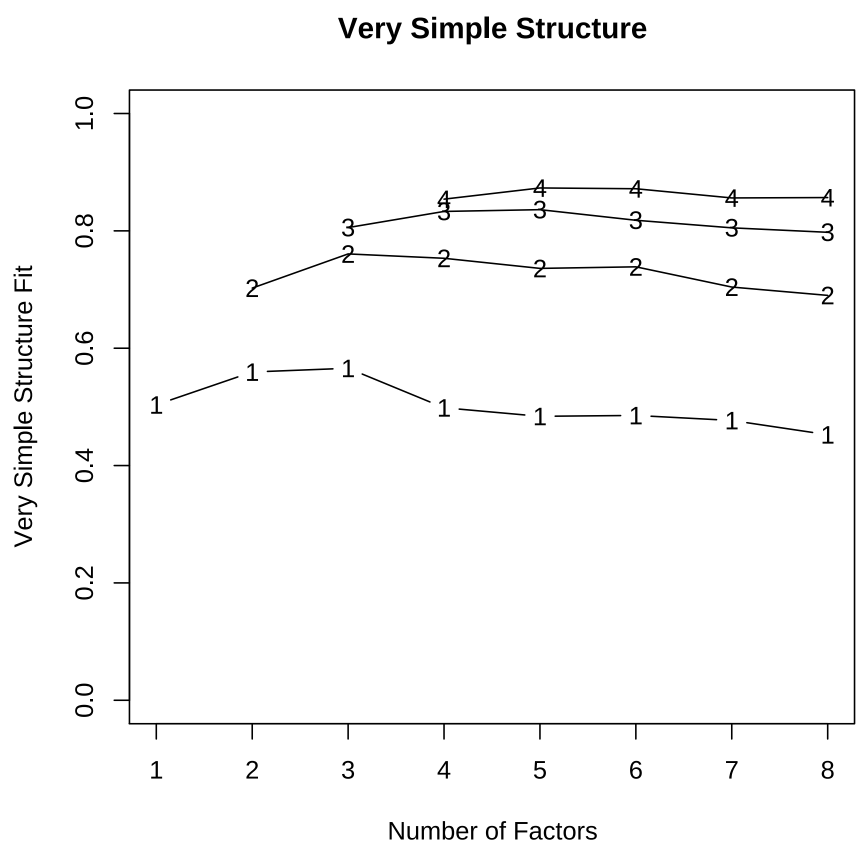

Figure 2: The Very Simple Structure criterion applied to the 30 facets of the NEO-PI-R. The most interpretable number of factors of complexity 1 or 2 is a 3 factor solution.

```
> neo.clus <- ICLUST(neo.df)
> ICLUST.rgraph(neo.clus, labels = neo.label[, ], min.size = 4)
```

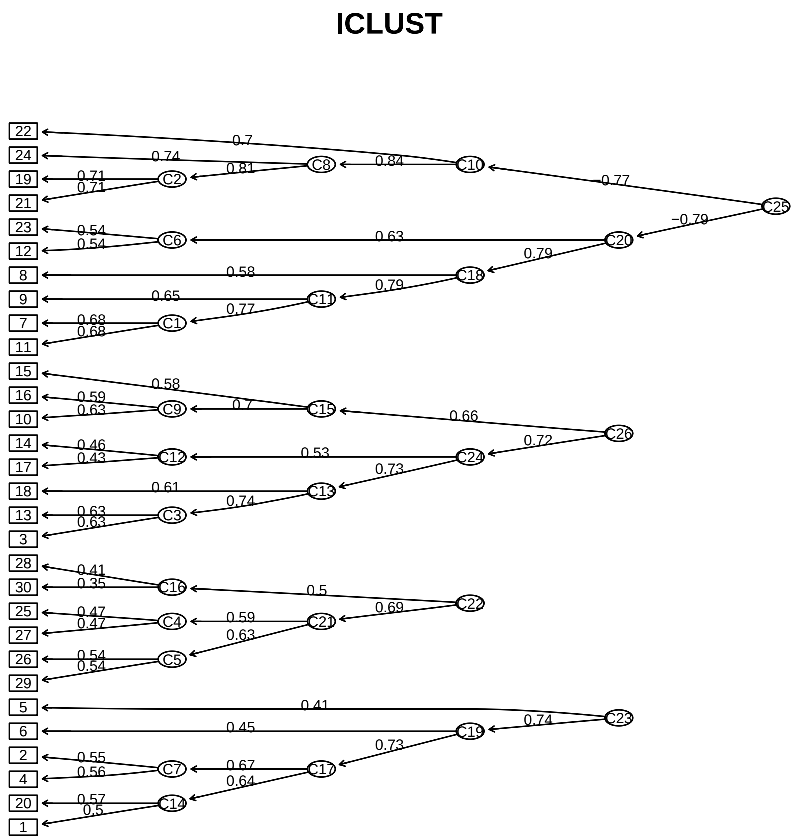

Figure 3: ICLUST analysis of the NEO

Table 2: The four cluster solution suggests that two of them are moderately intercorrelated.

```
> neo.clus$cor

      C22  C26   C23   C25
C22  1.00 0.41 -0.01 -0.02
C26  0.41 1.00  0.07  0.28
C23 -0.01 0.07  1.00  0.28
C25 -0.02 0.28  0.28  1.00
```



```
> neo.clus$cor

      C22  C26   C23   C25
C22  1.00 0.41 -0.01 -0.02
C26  0.41 1.00  0.07  0.28
C23 -0.01 0.07  1.00  0.28
C25 -0.02 0.28  0.28  1.00
```



```
> neo.clus <- ICLUST(neo.df, nclusters = 3, alpha = 0, beta = 0)
```

Cluster solutions are useful because they in fact represent the effect of unit weighted scoring. That is, while a five factor solution is frequently discussed, when actually forming unit weighted ocmposites of items or of facets, what is measured is not five independent factors but rather five highly intercorrelated scales. Using the simple concept that scales should be combined as long as the general factor (estimated in `ICLUST` by coefficient β) of the higher level scale is greater than that of either subcomponent, a three or four cluster solution is most appropriate.

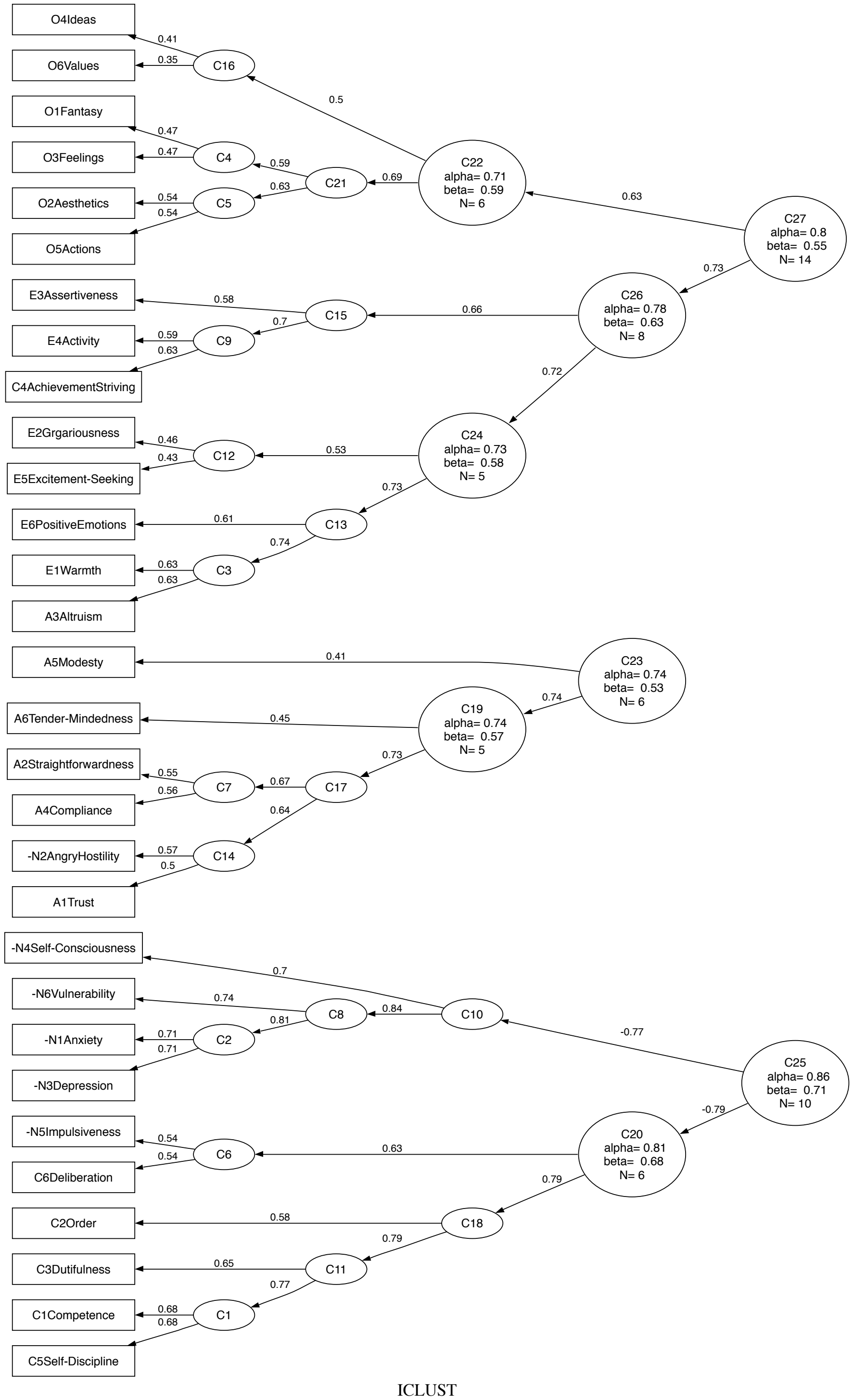

Figure 4: ICLUST analysis of the NEO set to extract 3 clusters

5 Interpreting the factors

Yet a final way to decide on the number of factors is factor interpretability. Here we compare 3, 4, and 5 factor solutions with a VARIMAX rotation to simple structure and the 5 factor solution with a oblimin transformation (using the *GPArotation* package).

Table 3: 3 and 4 factor solutions to the NEO PI- R

```
> neo3 <- factanal(covmat = neo, factors = 3)
> neo4 <- factanal(covmat = neo, factors = 4)
> neo34 <- cbind(neo3$loadings, neo4$loadings)
> round(neo34, 2)
```

	Factor1	Factor2	Factor3	Factor1	Factor2	Factor3	Factor4
N1Anxiety	-0.66	0.05	0.00	0.00	0.77	-0.11	-0.01
N2AngryHostility	-0.53	0.06	-0.46	0.00	0.59	-0.10	-0.49
N3Depression	-0.77	-0.05	-0.04	-0.08	0.78	-0.28	-0.05
N4Self-Consciousness	-0.64	-0.17	0.02	-0.20	0.68	-0.18	0.02
N5Impulsiveness	-0.54	0.29	-0.17	0.28	0.44	-0.28	-0.19
N6Vulnerability	-0.77	-0.18	0.02	-0.18	0.68	-0.39	0.02
E1Warmth	0.16	0.60	0.44	0.62	-0.10	0.14	0.41
E2Grgariousness	0.11	0.49	0.14	0.51	-0.12	0.04	0.12
E3Assertiveness	0.42	0.53	-0.24	0.50	-0.30	0.31	-0.27
E4Activity	0.21	0.57	-0.16	0.51	0.03	0.41	-0.21
E5Excitement-Seeking	-0.04	0.51	-0.28	0.49	0.02	-0.01	-0.30
E6PositiveEmotions	0.09	0.68	0.18	0.68	-0.04	0.12	0.14
O1Fantasy	-0.32	0.39	-0.14	0.42	0.15	-0.31	-0.15
O2Aesthetics	-0.04	0.37	0.13	0.38	0.06	0.02	0.10
O3Feelings	-0.21	0.61	0.02	0.58	0.29	0.05	-0.02
O4Ideas	0.10	0.39	0.03	0.42	-0.16	-0.03	0.02
O5Actions	0.17	0.36	-0.10	0.37	-0.17	0.06	-0.11
O6Values	-0.01	0.26	-0.07	0.29	-0.09	-0.12	-0.08
A1Trust	0.28	0.17	0.51	0.23	-0.31	0.06	0.51
A2Straightforwardness	0.17	-0.21	0.60	-0.20	-0.04	0.22	0.61
A3Altruism	0.22	0.37	0.59	0.37	-0.05	0.30	0.57
A4Compliance	0.13	-0.17	0.71	-0.11	-0.14	0.02	0.72
A5Modesty	-0.17	-0.24	0.50	-0.22	0.16	-0.07	0.50
A6Tender-Mindedness	-0.02	0.20	0.56	0.23	0.03	0.01	0.55
C1Competence	0.69	0.25	0.07	0.21	-0.40	0.61	0.06
C2Order	0.43	0.04	0.06	-0.05	-0.07	0.64	0.04
C3Dutifulness	0.54	0.03	0.28	-0.03	-0.22	0.61	0.28
C4AchievementStriving	0.48	0.35	-0.06	0.27	-0.10	0.69	-0.10
C5Self-Discipline	0.69	0.16	0.10	0.08	-0.32	0.76	0.08
C6Deliberation	0.49	-0.18	0.20	-0.23	-0.24	0.47	0.21

Table 4: Five factor solution to the NEO-PI-R.

```
> neo5 <- factanal(covmat = neo, factors = 5)

Call:
factanal(factors = 5, covmat = neo)

Uniquenesses:
            N1Anxiety      N2AngryHostility          N3Depression  N4Self-Consciousness
                0.399                 0.405                 0.304                 0.471
      N6Vulnerability              E1Warmth       E2Grgariousness       E3Assertiveness
                0.361                 0.405                 0.674                 0.501
 E5Excitement-Seeking    E6PositiveEmotions             O1Fantasy          O2Aesthetics
                0.621                 0.448                 0.593                 0.516
              O4Ideas             O5Actions              O6Values               A1Trust A2S
                0.722                 0.487                 0.847                 0.591
           A3Altruism          A4Compliance             A5Modesty    A6Tender-Mindedness
                0.388                 0.442                 0.677                 0.644
              C2Order         C3Dutifulness C4AchievementStriving     C5Self-Discipline
                0.586                 0.483                 0.413                 0.311

Loadings:
                      Factor1 Factor2 Factor3 Factor4 Factor5
N1Anxiety              0.766  -0.116
N2AngryHostility       0.594  -0.100  -0.478
N3Depression           0.783  -0.266          -0.106
N4Self-Consciousness   0.676  -0.179          -0.169
N5Impulsiveness        0.431  -0.310  -0.234   0.281
N6Vulnerability        0.677  -0.383          -0.158
E1Warmth              -0.116   0.116   0.324   0.648   0.205
E2Grgariousness       -0.142                   0.543
E3Assertiveness       -0.300   0.299  -0.318   0.399   0.246
E4Activity                     0.388  -0.278   0.479   0.170
E5Excitement-Seeking                  -0.378   0.464   0.141
E6PositiveEmotions                             0.706   0.203
O1Fantasy              0.161  -0.283  -0.151   0.170   0.499
O2Aesthetics                           0.143           0.664
O3Feelings             0.304                   0.402   0.463
O4Ideas               -0.155                   0.219   0.454
O5Actions             -0.152   0.127                   0.683
O6Values                                       0.112   0.343
A1Trust               -0.310           0.486   0.235   0.134
A2Straightforwardness          0.222   0.623          -0.113
A3Altruism                     0.255   0.501   0.538
A4Compliance          -0.140           0.732
A5Modesty              0.156           0.509          -0.162
A6Tender-Mindedness                    0.520   0.267   0.115
C1Competence          -0.399   0.614           0.181   0.117
C2Order                        0.619                  -0.146
C3Dutifulness         -0.209   0.626   0.285
C4AchievementStriving          0.699  -0.126   0.213   0.169
```

Table 5: Oblique transformations of the five factor solution indicate that factors one and two are highly correlated and factors four and five are moderately correlated.

	[,1]	[,2]	[,3]	[,4]	[,5]
[1,]	1.00	-0.41	-0.12	-0.13	-0.07
[2,]	-0.41	1.00	0.07	0.21	0.08
[3,]	-0.12	0.07	1.00	0.01	-0.16
[4,]	-0.13	0.21	0.01	1.00	0.32
[5,]	-0.07	0.08	-0.16	0.32	1.00

	Factor1	Factor2	Factor3	Factor4	Factor5
N1Anxiety	0.80	0.04	0.00	0.07	0.00
N2AngryHostility	0.58	0.05	-0.44	-0.12	0.01
N3Depression	0.78	-0.11	0.02	-0.04	0.05
N4Self-Consciousness	0.69	-0.03	0.10	-0.09	-0.05
N5Impulsiveness	0.37	-0.25	-0.28	0.30	-0.02
N6Vulnerability	0.65	-0.25	0.09	-0.06	-0.05
E1Warmth	-0.08	0.03	0.17	0.69	0.10
E2Grgariousness	-0.15	-0.07	-0.09	0.55	-0.01
E3Assertiveness	-0.29	0.23	-0.41	0.25	0.16
E4Activity	0.08	0.39	-0.38	0.36	0.09
E5Excitement-Seeking	-0.04	-0.07	-0.48	0.38	0.03
E6PositiveEmotions	-0.05	0.01	-0.12	0.70	0.08
O1Fantasy	0.09	-0.32	-0.14	0.11	0.47
O2Aesthetics	0.13	0.05	0.18	-0.01	0.70
O3Feelings	0.33	0.08	-0.10	0.34	0.42
O4Ideas	-0.17	-0.09	-0.01	0.15	0.43
O5Actions	-0.15	0.05	-0.03	-0.13	0.71
O6Values	-0.12	-0.16	-0.08	0.06	0.32
A1Trust	-0.28	-0.05	0.42	0.31	0.11
A2Straightforwardness	0.06	0.22	0.63	0.03	-0.06
A3Altruism	0.02	0.21	0.35	0.63	-0.11
A4Compliance	-0.08	-0.03	0.73	0.11	0.03
A5Modesty	0.20	-0.05	0.52	0.06	-0.12
A6Tender-Mindedness	0.07	-0.03	0.46	0.37	0.10
C1Competence	-0.29	0.56	-0.02	0.08	0.10
C2Order	0.06	0.66	0.00	0.00	-0.13
C3Dutifulness	-0.06	0.62	0.27	-0.03	0.05
C4AchievementStriving	0.04	0.72	-0.17	0.07	0.16
C5Self-Discipline	-0.18	0.73	-0.01	0.08	-0.08
C6Deliberation	-0.12	0.50	0.28	-0.23	0.02

6 Comparing solutions

It is possible to compare solutions by examining the factor congruence of the factor/clluster loadings.

```
> round(factor.congruence(neo4, neo5), 2)

        Factor1 Factor2 Factor3 Factor4 Factor5
Factor1   -0.16    0.22   -0.12    0.93    0.77
Factor2    1.00   -0.56   -0.22   -0.18   -0.10
Factor3   -0.56    1.00    0.19    0.32    0.04
Factor4   -0.24    0.22    0.99    0.15   -0.09

> round(factor.congruence(neo3, neo4), 2)

        Factor1 Factor2 Factor3 Factor4
Factor1    0.19   -0.91    0.84    0.27
Factor2    0.99   -0.13    0.30   -0.05
Factor3    0.04   -0.23    0.26    1.00

> round(factor.congruence(neo3, neo5), 2)

        Factor1 Factor2 Factor3 Factor4 Factor5
Factor1   -0.92    0.85    0.24    0.26    0.07
Factor2   -0.14    0.29   -0.16    0.93    0.75
Factor3   -0.24    0.25    0.98    0.21   -0.05
```

When comparing the 4 and 5 factor solutions, three are practically identical and the first factor of the 4 factor solution is a blend of the fourth and fifth factors. One factor (three) of the three factor solution corresponds with the third of of the five factor solution, and factor one of the three is a blend of factors one and two of the five and factor two is a blend of four and five.

What are these three factors?

Factor one is a combination of Conscientiousness and Emotional stability, perhaps interpreted as emotional and impulse control. Factor two of Extraversion and Openness reflects a general positive excitement to people and ideas, while factor three is the general agreeableness factor of trust in others and generally being nice.

References

Cattell, Raymond B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research*, 1(2):245–276.

Hofstee, Willem K., de Raad, Boele, and Goldberg, Lewis R. (1992). Integration of the big five and circumplex approaches to trait structure. *Journal of Personality and Social Psychology*, 63(1):146–163.

Horn, John L. and Engstrom, Robert (1979). Cattell's scree test in relation to bartlett's chi-square test and other observations on the number of factors problem. *Multivariate Behavioral Research*, 14(3):283–300.

Revelle, W. (1979). Hierarchical cluster-analysis and the internal structure of tests. *Multivariate Behavioral Research*, 14(1):57–74.

Revelle, W. and Rocklin, T. (1979). Very simple structure - alternative procedure for estimating the optimal number of interpretable factors. *Multivariate Behavioral Research*, 14(4):403–414.

Zinbarg, R. E., Revelle, W., Yovel, I., and Li, W. (2005). Cronbach's alpha, revelle's beta, and mcdonald's (omega h) : Their relations with each other and two alternative conceptualizations of reliability. *Psychometrika*, 70(1):123–133.